Journal of Philippine Development
Number 34, Volume XIX, No. 1, First Semester 1992

FUZZY SUBSET THEORY IN THE MEASUREMENT OF POVERTY *

M.C.S. Bantilan, F.T. Bantilan Jr. and M.M. de Castro *

INTRODUCTION

While the pervasive imprecision in the measurement of income is well known, researchers concerned with understanding the phenomenon of poverty and the welfare of society continue to rely heavily on measures of income based on household surveys. Alternative measures of welfare, however, are continuously being sought out. For example, expenditure is used to substitute for income as a measure of welfare. The subjective assessment of respondents regarding their own level of welfare in the so-called perception surveys has also gained popularity.

Four important criticisms on the traditional measures, as emphasized by Cerioli and Zani (1990), are reiterated. They are:

- Income data generated through interview responses are often times underestimated;
- Poverty is a multidimensional phenomenon so that complementary indicators reflecting poverty should be taken into account;
- There is a gradual transition from extreme poverty to wealth;
- Income itself is vague as a concept.

What has not been explored in traditional measures of poverty is the extensive set of categorical variables, indicating standard of living, long available from existing survey data. The difficulty of incorporating these

---

* Paper presented at the "Consultative Workshop on the Methodologies for an Integrated Analysis of Poverty and Income Distribution and Their Uses for Policy Formulation," SEAMEO-Innotech Building, Quezon City, 21-24 July 1992.

* Respectively, Associate Professors of the Department of Economics, College of Economics and Management and Institute of Mathematical Sciences and Physics, University of the Philippines at Los Baños; and Development Management Officer of the Development Planning and Research Project (DPRP), NPPS-National Economic and Development Authority (NEDA.

Financial support was provided by the United Nations Development Program. The authors deeply appreciate the encouragement of Dr. Camilo Dagum and the excellent assistance provided by Joanne M. Pattugalan and Rommel Ruby, also of the DPRP, NPPS-NEDA. The authors are entirely responsible for the form and content of this paper.

indicators in the traditional framework has precluded their use in deriving measures of poverty and welfare.

The theory of fuzzy subset provides a new approach to the use of traditional economic variables such as income or expenditure to derive new measures of poverty. Moreover, the approach can readily make use of the extensive information contained in the set of standard of living indicators. Instead of the all or nothing membership to the poor set, it allows for partial membership, whereby one does away with the conventional definition of the poverty line and takes into account the gradual transition from poverty to the state of wealth. In the incorporation of the information from the set of categorical variables available from household surveys, it takes into account the multi-dimensionality of the poverty phenomenon. The approach provides a formalism in the use of other variables in parallel and in complementation with income and expenditure.

The theory of fuzzy subset opens up new approaches in the analysis of income distribution, the refinement of the concept of poverty and its measures.

THEORETICAL CONSIDERATIONS

Definition of Fuzzy Subset

Idea of Fuzzy Subset

Much of our judgment depends on our ability to classify objects according to qualities or characteristics of interest. For example, we may classify objects according to size: with respect to the quality or characteristic of being small. To be concrete, let us consider the set of positive whole numbers — 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Let E denote the set, i.e.

$$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

And let A denote the subset of E characterized by the quality of being small.

According to ordinary set theory, each element of set E either belongs or does not belong to the subset A. This is expressed by the characteristic function $u_A(i)$, where

$$\begin{aligned} u_A(i) &= 1, \quad \text{if i belongs to A} \\ &= 0, \quad \text{if i does not belong to A} \end{aligned}$$

We may, for example, set the numbers 1, 2, 3, 4, 5 as belonging to the subset A; and the rest as not belonging to A. Ordinary set theory does not allow values between 0 and 1 for the characteristic function. One is forced to set up a sharp/abrupt dividing line between the quality of being small and not small. In our example, the sharp boundary lies between 5 and 6. In most cases, however, this sharp boundary is not realistic; the idea of being small is fuzzy.

Furthermore, the theory of fuzzy subset also allows values between 0 and 1. Thus, membership to a subset is no longer an all or nothing proposition. This idea of partial membership is more realistic — in consonance with our valuation of whether objects belong to a subset or not in accordance with the characteristics by which we classify them. Fuzzy subset theory is an extension of the ordinary set theory.

Going back to our example, let us assume that 1 is definitely small; and that 10 is definitely not small. In the framework of partial membership, it makes sense to assign a gradation of values to the elements of set E, representing their degree of belongingness to the subset A, decreasing as the number increases.

In this case, the generalized characteristic function now is called the membership function, $u_A(i)$, to the subset A, where

$$u_A(i) = x, \quad 0 \leq x \leq 1$$

where x is the value of the membership function for the i^{th} element in the closed interval between 0 and 1. To complete our example, we may set the values of the membership function $u_A(i)$ as follows:

$$\begin{array}{ll} u_A(1) = 1 & u_A(6) = 0.2 \\ u_A(2) = 0.9 & u_A(7) = 0.1 \\ u_A(3) = 0.8 & u_A(8) = 0 \\ u_A(4) = 0.6 & u_A(9) = 0 \\ u_A(5) = 0.4 & u_A(10) = 0 \end{array}$$

As designed, the value of the membership function decreases as the number increases away from 1. This is to conform with the reality of fuzzy membership to the set of being small.

Rigorous Definition due to Zadeh

We now take up the rigorous definition of a fuzzy subset due to Zadeh (1965). Let E be a set, where each element i is a full member, i.e. $u_E(i)$ is unity. Then a fuzzy subset A of E is a set of ordered pairs

$$\{i, u_A(i)\} \text{ for all elements of E}$$

where $u_A(i)$ is the degree of membership of i in A. If $u_A(i)$ takes its values in a set X, called the membership set, then $u_A(i)$ maps the set E to the set X, or i takes its values in X through the function $u_A(i)$. We write

$$i \xrightarrow[u_A(i)]{} X$$

The function $u_A(i)$ is likewise called the membership function. It is to be noted that the referential set E is an ordinary set, since each element in E has a full membership. The subset A, however, is a fuzzy subset, since some of its members have partial membership. That is why the theory is called theory of fuzzy subsets and not theory of fuzzy sets.

Notation

We now define the symbols we will use in the context of poverty studies. Let the following letters denote the corresponding entities indicated:

E	the referential set or the set of individuals or households in the population of interest;
i	the i^{th} element of set E;
C	a variable used to measure levels of welfare; C could be a continuous variable such as income; or a categorical variable, either dichotomous or polytomous;
C_j	the j^{th} variable in a set of k variables;
c_{ij}	the value of the j^{th} variable for the i^{th} element of set E;
A	the subset of E consisting of the poor;
$u_A(i)$	the membership function of the element i to the poor subset A;
x_{ij}	the value of the membership function $u_A(i)$ in the closed interval between 0 and 1, for the j^{th} variable and for the i^{th} element of set E.

Specification of the Membership Function

Determination of Critical Limits

In the study of poverty, the fuzzy subset approach requires the identification of an upper and lower bound, or specified characteristics to define three subsets of the population, namely:

a. the subset of the population who are *certainly poor* according to society's standard of well-being;
b. the subset of the population who are *certainly non-poor* according to society's standard of well-being;
c. the subset of the population who exhibit only partial membership to the poor set.

For a given variable C, let c_1 be the lower limit and c_2 the upper limit that divide the population into three such subsets. In the case of income, c_1 may be represented by an appropriately determined subsistence income; while c_2 may be represented by a measure of central tendency or a proportion of it. Using the notation above, we define

$$\mu_A = f(c)$$

where f(c) gives the grade of membership to the poor subset, in terms of the value of the variable C.

With the limits chosen, we have

$$u_A = 1 \quad \text{when} \quad c \leq c_1$$

$$u_A = 0 \quad \text{when} \quad c \geq c_2$$

$$u_A = f(c)\ , \quad 0 < f(c) < 1 \quad \text{when} \quad c_1 < c < c_2$$

Given a specified membership function f(c), the critical limits, c_1 and c_2, indicating the boundaries of the three subsets, are such that

$$\lim_{c \to c_1+} f(c) = 1$$

and

$$\lim_{c \to c_2-} f(c) = 0$$

The limits c_1 and c_2 define the region of transition from the state of extreme poverty to wealth.

Expressions of Some Membership Functions

The specification of the membership function $\mu_A(i)$ or f(c) is a basic step in the application of the fuzzy subset approach. An appropriate membership function may be defined in terms of the values of continuous variables (e.g., income and consumption) or in terms of values of the set of categorical variables characterizing the household.

In practical applications, the following membership functions with graphical illustrations shown in Fig. 1 to 7 may be considered.

(1) $f(c) = 1, \quad c \leq c_T$
$\quad\quad\; = 0, \quad c > c_T$

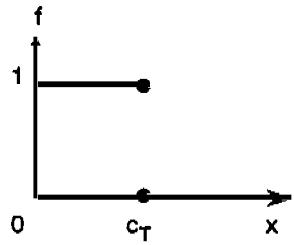


Fig. 1

(2) $f(c) = 1, \quad c \leq c_1$
$\quad\quad\; = \dfrac{c_2 - c}{c_2 - c_1}, \quad c_1 \leq c \leq c_2$
$\quad\quad\; = 0, \quad c \geq c_2$

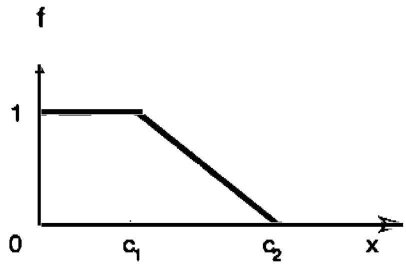


Fig. 2

(3) $f(c) = 1, \quad c \leq c_T$
$\quad\quad\; = e^{-k(c-c_T)}, \quad c > c_T$
and $k > 0$.

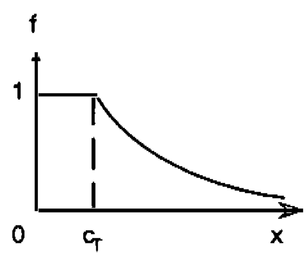


Fig. 3

(4) $f(c) = 1, \quad c \leq c_T$
$\quad\quad\; = e^{-k(c-c_T)^2}, \quad c > c_T$
and $k > 0$.

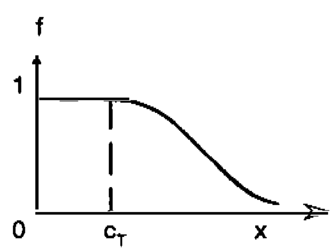


Fig. 4

$$(5)\quad f(c) = 1, \quad c \le c_T$$
$$= \frac{1}{1 + k(c - c_T)^2}, \quad c > c_T \text{ and } k > 0$$

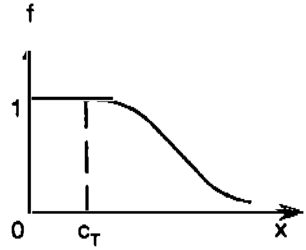


Fig. 5

$$(6)\quad f(c) = 1 - c_T c^k, \quad 0 \le c \le 1/^k\_a$$
$$= 0, \quad c \ge 1/^k\_a$$

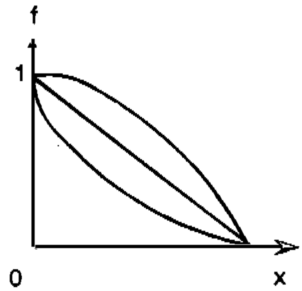


Fig. 6

$$(7)\quad f(c) = 1, \quad 0 \le c \le c_1$$
$$= \frac{1}{2} - \frac{1}{2}\mathrm{Sin}\frac{\pi}{c_2 - c_1}\left(x - \frac{c_1 + c_2}{2}\right)$$

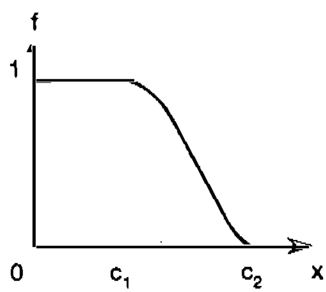


Fig. 7

In the equations above, f(c) defines the grade of membership to the poor set according to the value of c; c_T represents a specified poverty threshold; and c_1 and c_2 define the bounds of c separating the poor, the transition, and the nonpoor regions.

We now briefly indicate the usefulness of the functions listed above in defining the grade of membership of economic units to the poor subset. A membership function of the type given in equation (1) is handy in representing dichotomous household categories such as employment status, absence of savings, sex of household head, availability of electricity in the house, and ownership of radio, refrigerator, television and other basic household consumer durables. In this case, $u_A = 1$ identifies membership to the deprived subset of households; while $\mu_A = 0$ corresponds to non-membership to the deprived subset.

Appropriate membership functions for continuous variables and other categorical (non-dichotomous) variables such as quality of housing, tenureship status, source of water, availability/quality of toilet and sanitation, and the level of education of household members may be handled by using functions of the types given by equations (2) to (7).

Take for instance, the quality of housing materials symbolized by the variable C_2. The values of C_2 may be defined as follows:

c_2 = 1 for salvaged/makeshift materials;
= 2 for mixed but predominantly salvaged materials;
= 3 for light materials (cogon, nipa, anahaw);
= 4 for mixed but predominantly light materials;
= 5 for mixed but predominantly strong materials;
= 6 for strong materials (galvanized iron, aluminum, tile, concrete, brick stone, asbestos).

Using the membership function of type (2), and defining $c_1 = 2$ and $c_2 = 6$, then

f(c) = 1 for salvaged/makeshift materials;
= 1 for mixed but predominantly salvaged materials;
= .6 for light materials (cogon, nipa, anahaw);
= .4 for mixed but predominantly light materials;
= .2 for mixed but predominantly strong materials;
= 0 for strong materials (galvanized iron, aluminum, tile, concrete, brick stone, asbestos).

As another example, consider the level of per capita income of the Filipino household and define c_1 and c_2 as an appropriately measured subsistence income and mean per capita income of the reference population, respectively. For the Philippines in 1988, let us take c_1 to represent a level of subsistence threshold, say P 3,016, and c_2 equal to the mean income, say P 8,008. The corresponding membership function of type (7) is presented in Fig. 8. In effect, households suffering from severe deprivation or malnutrition are considered completely poor; those having incomes higher than the average would be non-poor; while those with income between c_1 and c_2 are assigned gradually decreasing grades of membership to the poor subset.

Variations of the functions of the types (1) to (7) may of course be applied to determine appropriate grades of membership to the poor subset.

Sampling Scheme Suggested by the Fuzzy Subset Approach

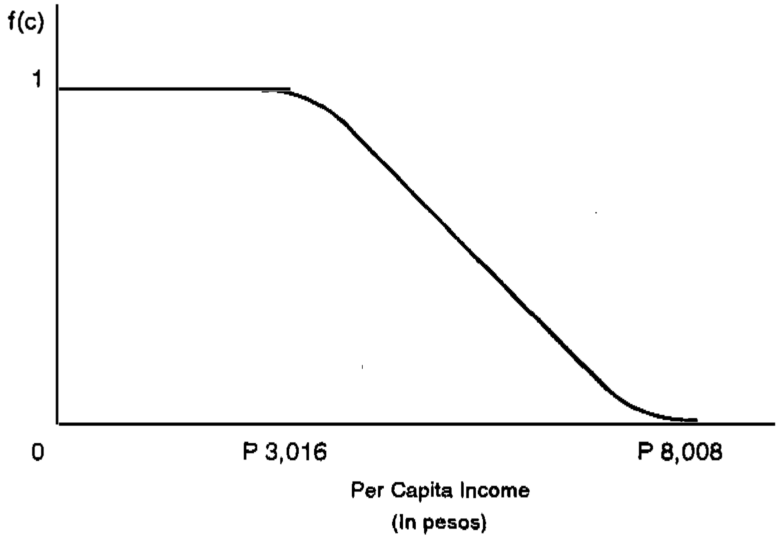
If the purpose is to derive poverty measures for a given population, a cost-effective sampling scheme is suggested by the fuzzy subset methodology. This scheme comprises two stages, namely: (1) a sample of the reference population representative of all sectors including both the poor and the nonpoor is taken and questions relating only to basic structural characteristics are asked; and (2) the group of households at risk of being poor is derived from the first-stage sample and on which detailed information about income, expenditure and categorical variables reflecting their level of welfare are obtained.

This approach provides a less costly way of obtaining the required information about the sector of interest, i.e. those who are at risk of being poor, and subsequently of computing the grade of membership of each member in the poor subset. Those identified as definitely non-poor in the first stage are automatically assigned a grade of membership equal to zero.

The fuzzy subset approach has a clear advantage.

The data requirements of the fuzzy subset approach is less stringent than the requirements of the more often used income distribution modelling approach. In the latter approach, responses on income, expenditures and other variables are required from all sectors of the reference population. The undercoverage and underreporting occurring with respect to responses from the non-poor sector, particularly the very wealthy households, have been identified as sources of inaccuracy of measures derived from income distribution modeling efforts which uses probability density functions of income or other observable variables.

Figure 8
A SAMPLE MEMBERSHIP FUNCTION



EVALUATION OF INDIVIDUAL DEPRIVATION

Alternative Data Sources

Alternative data sources for welfare measures are available for evaluating the grade of deprivation of a unit in the reference population. Among these are income, consumption expenditures and standard of living indicators. Income and expenditure data are available from standard surveys regularly conducted by government survey bureaus. For example, the Family Income and Expenditure Surveys (FIES) conducted by the National Statistics Office for more than three decades now provide these types of information. The survey data contains a rich collection of variables, both continuous and categorical, which indicates the household's level of welfare. Income, expenditures and savings are among the continuous variables available. Categorical data describing the household include type of dwelling unit, source of water supply, presence of electricity in the house, absence or presence of savings, sanitation facilities, ownership of radio or ownership of car, etc.

The evaluation of the membership function through subjective judgments of the individuals or households themselves (Van Praag 1971 or Hagenaars 1986) may also be utilized. Data available from the series of perception surveys conducted by the Social Weather Station of the Philippines, for example, give us the subjective judgments of individuals about their own perception of their level of welfare. Interviewees' responses to the question "Mahirap? o Hindi mahirap?" and their answers regarding where they think they are in the poor-not poor range are available.

Measurement of Individual Deprivation

A measure of individual deprivation is the value of the membership function $u_A(i)$. The evaluation of the membership function maps the economic unit i via the function f(c) onto the closed interval [0,1].

Let there be k variables C_1, C_2, ... C_k that describe the set E of n individuals. With the critical limits, c_1 and c_2, appropriately chosen for each variable, define

$$
\begin{aligned}
x_{ij} &= f(c_{ij}) \quad \text{for } c_1 < c_{ij} < c_2 \\
&= 1 \quad \text{for } c_{ij} \leq c_1 \\
&= 0 \quad \text{for } c_{ij} \geq c_2
\end{aligned}
$$

where x_{ij} is the value of the membership function for the i^{th} individual and for the j^{th} variable. A measure of individual deprivation is then derived by computing the weighted average across the k variables, i.e.,

$$\mu_A(i) = \sum_{j=1}^{k} x_{ij} w_j / \sum_{j=1}^{k} w_j \tag{8}$$

where w_j is a weight appropriately chosen for each variable C_j, reflecting its level of importance. If equal weights are assumed, then the membership function reduces to

$$\mu_A(i) = 1/k \sum_{j=1}^{k} x_{ij}$$

Examples

This section presents applications of the fuzzy subset approach to sample data collected in the Philippines in 1988 through the FIES. Responses from a random sample consisting of 18,922 households, representing a total of 10,533,927 households, were obtained through interviews. This data set is utilized in the following examples.

Using Categorical Variables

Seven categorical variables are chosen for this example to describe how various aspects of the level of welfare of households are taken into account in parallel. Philippine data from the FIES survey of 1988 is used. Another paper entitled "Application of Fuzzy Subset Theory in the Measurement of Poverty in the Philippines" considers a more comprehensive set of variables from the FIES data set. The variables chosen for this example includes:

C_1 = Sanitation or type of toilet facilities (SANITATION);
C_2 = Walling materials of dwelling unit (WALL);
C_3 = Presence of electricity in the house (ELEC);
C_4 = Source of Water Supply (WATER);
C_5 = Absence of Savings (ABSAVE);
C_6 = Ownership of Radio (RADIO);
C_7 = Ownership of Car (CAR).

Table 1 defines the codes describing each variable including the specification of the limits, c_1 and c_2. Application of equation (2) yields estimates of the extent of deprivation for each household with respect to each variable j, j = 1, ..., 7.

Weights, denoted by v_j, equal to the logarithm of the inverse rate of households showing the corresponding symptom of deprivation is applied to each variable, that is,

$$v_j = \log [n_j / M_j],\ j = 1, \ldots, n$$

$$M_j = \sum_{i=1}^{n} x_{ij}$$

where M_j refers to the sum over all households of the degree of deprivation with respect to the attribute or variable j; and n_j is the total number of households for which the variable j is observed. In this example, all seven variables are applicable among the population of households. Thus, n_j is fixed at 10,533,928. Table 2 illustrates the frequency of occurrence (M_j) of these indicators for possible membership to the poor subset. Presented in the last column of Table 2 are the normalized weights, denoted by w_j, computed by taking the relative percentage of v_j with respect to its total, i.e.,

$$w_j = \frac{v_j}{\sum_{j=1}^{k} v_j}$$

It is observed that the weighting system developed attaches greatest weight to basic necessities compared to the other attributes (variables) not widely available to the members of the reference population. In this

Table 1
CODE DESCRIPTIONS OF VARIABLES SELECTED FROM THE FIES SURVEY DATA, 1988

Variables	Description	Codes	Limits
C1= SANITATION	Kind of toilet Facilities		
	None	1	c_1=1
	Others (pail system)	1	c_2=5
	Open pit	2	
	Close pit	3	
	Water sealed	5	
C2= WALL	Walling materials of dwelling unit		
	Salvaged/makeshift materials	1	c_1=1
	Mixed but predominantly salvaged materials	1	
	Light materials	2	c_2=3
	Mixed but predominantly light materials	2	
	Mixed but predominantly strong materials	3	
	Strong materials	3	
C3= ELEC	Presence of electricity in the house		
	No	1	c_1
	Yes	2	c_2
C4= WATER	Source of water supply		
	Spring, river, stream, etc.	1	c_1=1
	Dug well	1	c_2=6
	Rain	1	
	Peddler	1	
	Tubed, piped well, owned by others	3	
	Faucet, owned by others community water system	4	

Table 1 *(continued)*

Variables	Description	Codes	Limits
	Tubed, piped well, own use	5	
	Faucet inside the house/ yard, community water system	6	
C5= ABSAVE	Absence of savings		
	Savings <= 0	1	c_1=1
	Savings > 0	2	c_2=2
C6= RADIO	Ownership of radio		
	No	1	c_1=1
	Yes	2	c_2=2
C7= CAR	Ownership of car		
	No	1	c_1=1
	Yes	2	c_2=2

example, the variable C_2 (type of materials of dwelling unit) is assigned greater weight than C_7 (ownership of car).

Applying equation (8) to each household, we obtain an estimate of the amount of individual deprivation. A summary of the results, shown in Table 3, indicates that the membership function ranges from 0 to 1. It also indicates that about 47 percent of the population have rates of deprivation higher than the average.

Fuzzy Graph for a Two-Variable Case

The fuzzy subset concept is illustrated for the two-variable case. Two variables are considered, namely: (1) type of materials used for the dwelling unit (HOUSING); and (2) source of water (WATER). Table 4 contains the weighted averages of the probability of membership in the poor set considering the two variables chosen. The gradation of shades shown in the accompanying fuzzy graph (Fig. 9) approximates the possible varying degrees of membership to the poor set of the reference population.

Using Single Continuous Variables

To evaluate the membership function to the poor set, we use equation (2) for the income and expenditure data. Fixing c_1 equal to P 3,015.80 (subsistence income for 1988 determined by the Technical Working Group

Table 2
FREQUENCY OF OCCURRENCE OF POVERTY SYMPTOMS CORRESPONDING TO SEVEN SELECTED INDICATORS OF DEPRIVATION AMONG FILIPINO HOUSEHOLDS

	Frequency of Occurrence,[1](Mj)	% of Total Population	vj	wj
1. WALL	2,562,857	.24	.61387	.2122
2. SANITATION	3,754,222	.35	.44807	.1549
3. ELEC	4,255,416	.40	.39672	.1372
4. WATER	4,986,613	.47	.32478	.1123
5. ABSAVE	2,904,625	.28	.55950	.1934
6. RADIO	3,146,277	.30	.52479	.1814
7. CAR	9,945,847	.94	.02495	.0086
		Total	**2.89268**	**1.0**

[1] The frequency distribution is drawn from a sample of 18,922 households. Weights for each sampling unit, developed for the FIES survey, is applied. Thus, the total population of Filipino households in 1988 (i.e., n = 10,533,928) is reflected in the above table.

Table 3
FREQUENCY OF THE GRADE OF DEPRIVATION BASED ON SELECTED CATEGORICAL VARIABLES PHILIPPINES, 1988

$\mu_A(i)$	Frequency	%	Cumulative Percentage
$\mu_A(i) = 1$	4466.	0.0	0.0
$.875 <= \mu_A(i) <= 1$	112621.4	1.1	1.1
$.75 <= \mu_A(i) <= .875$	300123.1	2.8	4.0
$.625 <= \mu_A(i) <= .75$	1174788.2	11.2	15.1
$.5 <= \mu_A(i) <= .625$	1196477.4	11.4	26.5
$.375 <= \mu_A(i) <= .5$	1607043.8	15.3	41.7
$.25 <= \mu_A(i) <= .375$	1583621.5	15.0	56.8
$.125 <= \mu_A(i) <= .25$	2099691.5	19.9	76.7
$0 <= \mu_A(i) <= .125$	2260242.8	21.5	98.2
$\mu_A(i) = 0$	194851.1	1.8	100.0

Quantiles:

$Q_{100} = 1$	$Q_{75} = 0.485827$	Mean = .332172
$Q_{99} = 0.864305$	$Q_{50} = 0.276537$	Median = .276537
$Q_{95} = 0.715514$	$Q_{25} = 0.113519$	Mode = .008534
$Q_{90} = 0.654975$		

Table 4
WEIGHTED AVERAGES OF THE PROBABILITY OF MEMBERSHIP IN THE POOR SET: A TWO-VARIABLE CASE

HOUSING (fj=.61387; wj=.654)

NP

0	3	.346	.277	.208	.138	.069	0
0.5	2	.673	.604	.535	.465	.396	.327
1	1	1	.931	.862	.792	.723	.654
	Cj	1	2	3	4	5	6
Xj		1	0.8	0.6	0.4	0.2	0

P NP

WATER SUPPLY (fj=.32478; wj=.346)

Fig. 9
FUZZY GRAPH FOR THE CASE OF TWO CATEGORICAL VARIABLES

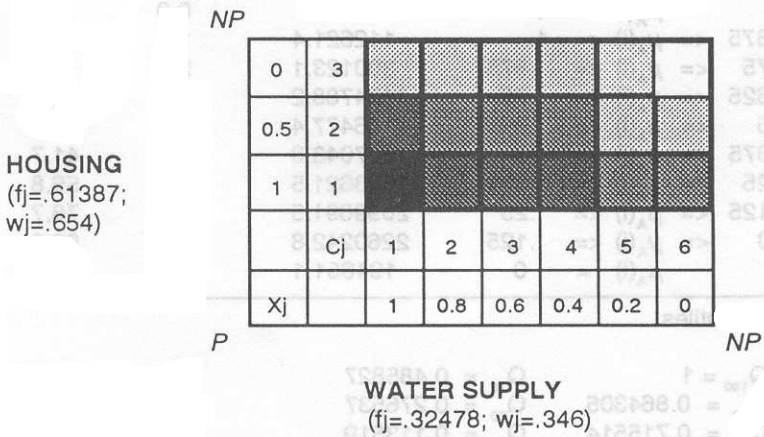


Table 5
FREQUENCY DISTRIBUTION OF GRADES OF MEMBERSHIP IN THE POOR SET AMONG FILIPINO FAMILIES BASED ON PER CAPITA INCOME, 1988

$\mu_A(i)$				Frequency	%	Cumulative Percentage
		$\mu_A(i)$	= 1	2109484.5	20.0	20.0
.875	<=	$\mu_A(i)$	<= 1	1026308.8	9.8	29.8
.75	<=	$\mu_A(i)$	<= .875	931581.2	8.8	38.6
.625	<=	$\mu_A(i)$	<= .75	803400.9	7.6	46.2
.5	<=	$\mu_A(i)$	<= .625	696591.6	6.6	52.8
.375	<=	$\mu_A(i)$	<= .5	564130.2	5.4	58.2
.25	<=	$\mu_A(i)$	<= .375	486930.2	4.6	62.8
.125	<=	$\mu_A(i)$	<= .25	421329.7	4.0	66.8
0	<=	$\mu_A(i)$	<= .125	38883.5	3.7	70.5
		$\mu_A(i)$	= 0	3105286.2	29.5	100.0

Mean = .501263
Median = .502428

Table 6
FREQUENCY DISTRIBUTION OF GRADES OF MEMBERSHIP IN THE POOR SET AMONG FILIPINO FAMILIES BASED ON PER CAPITA EXPENDITURES, 1988

$\mu_A(i)$				Frequency	%	Cumulative Percentage
		$\mu_A(i)$	= 1	2531767.2	24.0	24.0
.875	<=	$\mu_A(i)$	<= 1	896309.3	8.5	32.5
.75	<=	$\mu_A(i)$	<= .875	776691.4	7.4	39.9
.625	<=	$\mu_A(i)$	<= .75	695332.5	6.6	46.5
.5	<=	$\mu_A(i)$	<= .625	603938.9	5.7	52.3
.375	<=	$\mu_A(i)$	<= .5	528292.1	5.0	57.3
.25	<=	$\mu_A(i)$	<= .375	427105.0	4.1	61.3
.125	<=	$\mu_A(i)$	<= .25	415760.8	3.9	65.3
0	<=	$\mu_A(i)$	<= .125	352534.1	3.3	68.6
		$\mu_A(i)$	= 0	3306195.5	31.4	100.0

Mean = .502668
Median = .487672

on Poverty of the National Statistical Coordination Board (NSCB) of the Philippines and c_2 equal to P 8,008.40 (mean per capita income), we have

$$\mu_A(i) = 1, \quad 0 \leq c < c_1$$
$$= 0, \quad c > c_2$$

For per capita incomes between P 3,015.80 and P 8,008.40, the membership function takes on values in the [0,1] interval where

$$\mu_A(i) = 1, \quad c < c_1$$
$$= \{c_2 - c\}/\{c_2 - c_1\}, \quad c_1 < c < c_2$$
$$= 0, \quad c \geq c_2$$

Table 5 summarizes the frequency distribution of $\mu_A(i)$ based on per capita income while Table 6 contains the results based on per capita expenditures. Fig. 10 is a fuzzy graph illustrating the distribution of continuous variables, e.g., per capita income and per capita expenditure. As shown, the gradual transition from the state of being definitely poor to the state of being definitely not poor is approximated by the gradation of shades from pure black to pure white. The average level of deprivation derived from both data sets is 50 percent. The next section will show how the above quantification of individual deprivation may be utilized to derive a measure of the extent of poverty of the total population.

POVERTY MEASURES: EVALUATION OF POPULATION DEPRIVATION

Formulation of Poverty Measures

A derivation of a measure of the extent of poverty of the total population is as follows:

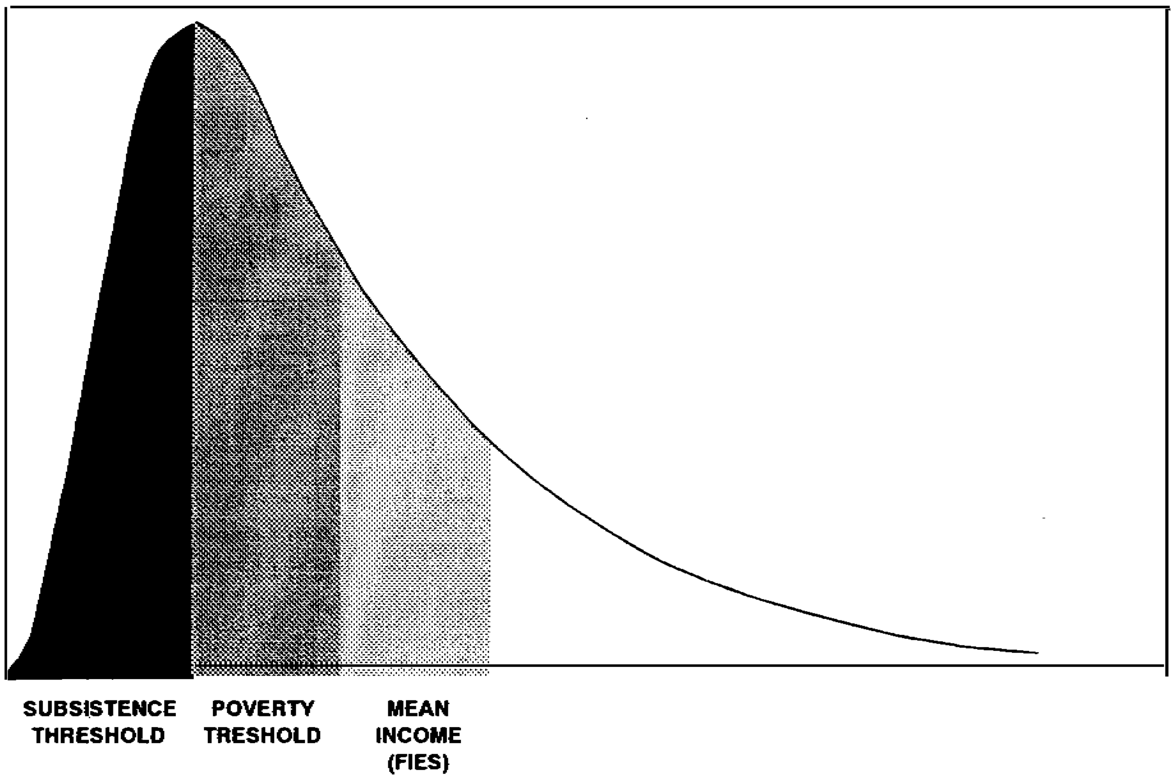
Let n be the number of individuals in a population. Then,

$$E = \{1, 2, ..., n\}.$$

A generalized formulation for measures of poverty for this population can be expressed as an aggregation of individual deprivation levels where the sum over all members of the population is taken, i.e., the cardinality of the fuzzy subset A of the poor

$$D = |A| = \sum_{1=i}^{n} \mu_A(i).$$

Fig. 10
FUZZY GRAPH FOR PER CAPITA INCOME PDF
PHILIPPINES, 1988



The cardinality |A| is a natural extension of the traditional concept of set cardinality (Dubois and Prade, 1980). In other words, the overall extent of the deprivation of population of size n may be measured by taking the totality of the levels of deprivation of all units in the population.

A coefficient reflecting society's aversion to the state of poverty may be desired and incorporated through a coefficient of aversion, say α, to obtain

$$D = \sum_{1=i}^{n} [\mu_A(i)]^{\alpha}$$

Subsequently, a generalized expression of the measure of poverty incidence is obtained by taking the average of the individual deprivation levels, i.e.,

$$P = 1/n \sum_{i=1}^{n} [\mu_A(i)]^{\alpha} \tag{9}$$

The measure P indicates the proportion of the population belonging, in a fuzzy sense, to the poor subset, where P ranges from 0 to 1. In the extreme case where all members belong to the non-poor subset with certainty, then $\mu_A(i) = 0$ for all i = 1, ..., n; and P = 0 (case of absence of poverty). On the other hand, if all $\mu_A(i) = 1$ for all i, then P = 1 (case of extreme poverty for all).

Substituting equation (8) for $\mu_A(i)$,

$$P = 1/n \sum_{i=1}^{n} [\sum_{j=1}^{k} \frac{x_{ij} w_j}{\sum_{j=1}^{k} w_j}]^{\alpha} \tag{10}$$

where k denotes the number of variables available to describe the level of individual welfare for each household i.

The generalized formulation, equation (9), takes on a specific form with the choice of a particular expression of the function $\mu_A(i)$. Let us consider the case K = 1. For instance, take equation (2) so that equation (9) becomes

$$P = 1/n \sum_{i=1}^{q} [\frac{c_2 - c_i}{c_2 - c_1}]^{\alpha}, \quad c_1 \leq c_i \leq c_2 \tag{10}$$

where $\mu_A(i) = 0$ when $c_i > c_2$; $\mu_A(i) = 1$ when $c_i < c_1$; and q is the number of households at risk of poverty or where $c_i < c_2$. With this choice we derive the following measures of poverty.

Case 1. Let $\alpha = 1$ to obtain

(11) $$P = 1/n \sum_{i=1}^{q} \mu_a(i) = 1/n \sum_{i=1}^{q} \left(\frac{c_2 - c_i}{c_2 - c_1} \right), c_1 < c_i < c_2$$

The measure given by equation 11 reduces to the traditional headcount ratio when c_2 is evaluated at some specified threshold level. The traditional ratio is one where each individual is identified as poor with certainty ($\mu = 1$) or nonpoor with certainty ($\mu = 0$) depending on its location relative to an identified poverty threshold. In the case indicated by equation (11), the limit of μ_A (i) is equal to 1 as c_i approaches c_1; and the limit of μ_A is equal to 0 as c_i approaches c_2. Table 7 illustrates the observation that as the two limits, c_1 and c_2, approach the reference threshold level, say c_T, then the traditional headcount ratio is obtained.

Case 2. For $\alpha = 2$, then

$$P = 1/n \sum_{i=1}^{q} \left(\frac{c_2 - c_i}{c_2 - c_1} \right)^2, \quad c_1 < c_i < c_2.$$

where $\mu_A(i) = 0$ when $c_i > c_2$; $\mu_A(i) = 1$ when $c_i < c_1$.

For $\alpha = 2$ and the lower bound is equal to 0, then

$$P = 1/n \sum_{i=1}^{q} \left(\frac{c_2 - c_i}{c_2} \right)^2$$

The above expression is equivalent to another common measure of poverty developed by Forster, Greer and Thornbecke (FGT).

Several other special cases of equation (9) may be specified by taking other functional forms for $\mu_A(i)$ and various levels of the coefficient of aversion α.

Table 7
ESTIMATES OF POVERTY INCIDENCE BASED ON THE FUZZY SUBSET APPROACH CONSIDERING +/- STANDARD DEVIATION BANDS ABOUT THE POVERTY THRESHOLD

	HIGH					
PT	**5951.125 (+.0001)**	**6231.258 (+.001)**	**7476.292 (+.005)**	**8098.808 (+.007)**	**8876.954 (+.0095)**	**9032.584 (+.01)**
5888.874 (-.0001)	0.53939					
5608.741 (-0.001)		0.53929				
4363.708 (-0.005)			0.53094			
3741.191 (-0.007)				0.52312		
2963.045 (-.0095)					0.51027	
2807.416 (-0.01)						0.50737

ST = 3015.8
PT = 5920
Mean (Dagum) = 9257
SD = 311258.4
HR = 54.1

Some Results Using Philippine FIES Data

❑ ***Case where k = 1***: Fuzzy Subset Measures of Poverty Based on Income and Expenditure, 1988.

This section presents the results of poverty measurement based on two continuous variables, namely income and expenditure. Table 8 presents results based on per capita income and expenditure data.

The values contained in these tables are computed using equation 11, i.e.,

Table 8
ESTIMATES OF POVERTY INCIDENCE BASED ON THE FUZZY SUBSET MEASUREMENT USING FIES PER CAPITA INCOME AND EXPENDITURE DATA, 1988

	c_2 = Upper Limit			
	INCOME		EXPENDITURE	
Statistics	1988 Mean Per Capita Income (P 8603)	1988 Mean PCE (P 9516)	1988 Mean Per Capita Expenditure (P 6554)	1988 Mean PCE (P 9516)
c_1 = Lower Limit				
Subsistence Level (TWG) (P 3016)	.50126	.53488	.48558	.60466
Mode (Dagum) (P 3156/a, P 3017/b)	.50875	.54204	.48566	.60472
3rd Decile (Dagum) (P 3763/a, (P 3400/b)	.53961	.57152	.46579	.62535
4th Decile (Dagum) (P 4559/a, P 4045/b)	.57594	.60614	.55016	.65666
Median (Dagum) (P 5490/a, P 6407/b)	.61288	.64124	.58930	.68801

a/ Value of income.
b/ Value of expenditure.

$$P = 1/n \sum_{i=1}^{q} \mu_A(i) = 1/n \sum_{i=1}^{q} \left(\frac{c_2 - c_i}{c_2 - c_1} \right)$$

As shown, alternative critical bounds (c_1 and c_2) are considered in the analysis. The lower limit, c_1, is given by 5 levels, namely: (a) subsistence threshold determined by the TWG of NSCB; (b) the mode of the income distribution; (c) the third decile; (d) fourth decile; and (e) the median based on the estimated Dagum model (Bantilan *et al*, 1991). The upper bounds took either the mean per capita income (based on the FIES data) and the

Table 9
POVERTY MEASURES BASED ON A SET OF SELECTED CATEGORICAL VARIABLES, AND PER CAPITA INCOME AND EXPENDITURES DATA: PHILIPPINES, 1988

Data	Approach		
	FST Approach	Modelling Approach	Published Measures (NSO)
I. Categorical Variables (Example 1)	.3322		
II. Per Capita Income	.5013	.541	.552
III. Per Capita Expenditure	.5027	.621	.650

mean per capita personal consumption expenditure based on published National Income Account estimates. These limits are given in pesos and indicated in parenthesis.

Taking the subsistence threshold determined by the TWG of NSCB as lower limit (c_1) and the mean per capita income as upper limit (c_2), poverty incidence is measured at 50.1 percent. The absolute magnitudes of the poverty incidence rates based on per capita expenditure is estimated at 48.6 percent.

Applying alternative lower limits i.e., mode, 3rd, 4th decile and the median of the income distribution) gave poverty incidence rates of 51-61 percent which is consistently a little higher than the estimates based on corresponding expenditure data (i.e., 49-59 percent) in 1988. An interesting observation is the consistency of the results irrespective of the limits used.

❑ ***Case where k = 7***: Fuzzy Subset Measures Based on a Set of Selected Categorical Variables: Philippines

Table 9 presents a measure of population deprivation in 1988 based on the Fuzzy Subset Theory (FST) Approach using the seven selected attributes of the previous example. A summary is presented comparing the results derived with those obtained using income and expenditure data. A comparison with other traditional measures is also provided. The higher level of estimates of poverty incidence drawn from the use of income data compared to those obtained by utilizing categorical variables may be indicative of possible underreporting of income in the FIES Survey.

SUMMARY AND CONCLUSION

The application of the concept of fuzzy subsets in the measurement of poverty provides great potential in improving the current methodologies in measuring levels of individual deprivation as well as the overall level of deprivation of society.

The traditional measures, e.g., the headcount ratio, requires a specified cutoff income (e.g., poverty threshold) to separate the poor from the nonpoor. The improvement over current procedures relates to three aspects. *First*, the gradual transition from extreme poverty to wealth is accounted for. The measurement of individual deprivation takes different grades of membership to the poor sector depending on the observed attributes like income or expenditures. *Second*, the use of fuzzy subsets and corresponding membership functions for a set of attributes allows us to synthesize alternative fuzzy relations and takes into account the overall level of welfare of the household. The multidimentionality nature of poverty which income and expenditure alone may not capture is accounted for. *Third*, the traditional income-based indices may result in incorrect findings as respondents usually provide imprecise information about their income. An evaluation of the level of deprivation of the household and society is enhanced by considering other observed characteristics which may more accurately describe the households' state of welfare.

REFERENCES

Cerioli, A. and Zani, S. "A Fuzzy Approach to the Measurement of Poverty." In *Income and Wealth Distribution, Inequality and Poverty (Studies in Contemporary Economics).* C. Dagum and M. Zenga (eds) Berlin, Heidelberg: Springer-Verlag, 1990.

Dubois, D. and Prade, H. *Fuzzy Sets and Systems: Theory and Applications.* New York: Academic Press, 1980.

French, S. " Fuzzy Decision Analysis: Some Criticism." In *Fuzzy Sets and Decision Analysis* H.J.L. Zimmermann, L.A. Zadeh and B.R. Gaines (eds). Amsterdam: North-Holland, 1984.

Gaines, B.R. "Foundations of Fuzzy Reasoning" In *Fuzzy Automata and Decision Processes* eds. M.M. Gupta, G.N. Saridis and B.R. Gaines (eds.) New York: North-Holland, 1977.

Giles, R. "The Concept of Grade of Membership" *Fuzzy Sets and Systems* 25 (1988): 297-323.

Hagenaars, A.J.M. *The Perception of Poverty.* Amsterdam: North-Holland, 1986.

Hisdal, E. "Are Grades of Membership Probabilities?" *Fuzzy Sets and Systems* 25 (1988): 325-348.

Kaufman, A. *Introduction to the Theory of Fuzzy Subsets.* Vol. 1. *Fundamental Theoretical Elements*; Vol. 2. *Application* (Information and Decision Processes in Humans and Machines). New York: Academic Press, 1975.

Van Praag, B.M.S. "The Welfare Function of Income in Belgium: An Empirical Investigation." *European Economic Review* 2 (1971): 337-369.

Zadeh, L.A. "Fuzzy Sets, Information and Control" *Journal of Mathematical Analysis and Applications* 8 (1965): 338-353.

__________. "Probability Measures of Fuzzy Events" *Journal of Mathematical Analysis and Applications* 23 (1968): 421-427.